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The spin structure of the constituent quarks and of the nucleon

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Abstract. We define a constituent quark within QCD. It is shown that the spin of such a quark and hence also the spin of the nucleon is a complex phenomenon. We discuss it in view of the experimental data.

Experiments have revealed that the internal structure of the nucleon is more complicated than originally assumed [1,4]. In particular, the portion of the nucleon spin carried by the spins of the u- and d-quarks is not, as naively expected, about 75%, but much smaller. In this paper we study the situation by considering the internal spin structure of the constituent quarks.

Often it is assumed that the nuleon consists of three constituent quarks, each of them having its own internal structure. However, in QCD the notion of a constituent quark has remained vague. Using a specific "gedankenexperiment" we first show how a constituent quark can be defined with all its dynamical properties. Then we apply our results to the nucleon. In order to simplify the task, we shall first neglect the s-quarks. Later on, we comment on what changes once s-quarks are introduced.

We relate the spin structure to the QCD anomaly [4], and a very specific picture emerges. For a proton being a system of three constituent quarks, i.e. (uud), it is difficult to disentangle the contributions of the three constituent quarks. Therefore we consider a heavy baryon of spin 3/2, e.g. one with the quark structure (bbu). The ground state of such a system is an isospin doublet, which we denote as U and D:

$$\begin{aligned} U \Uparrow &= (b \Uparrow b \Uparrow u \Uparrow), \\ D \Uparrow &= (b \Uparrow b \Uparrow d \Uparrow). \end{aligned} \tag{1}$$

One expects that these states of spin 3/2 exist in reality but will probably never be observed and studied in detail. We proceed to study the internal structure of these states, in particular the aspects related to the light quarks. Note that a state like (bbu) consists of a single light constituent quark.

The states U and D behave much like the proton–neutron system, if we turn off the weak interaction of the b-quarks. For example, the state D, being slightly heavier than U, would show β-decay: $D \to U + e^- + \bar{\nu}_e$. This can be used to define the associated vector and axial-vector coupling constants, e.g. the matrix elements $\langle U|\bar{u}\gamma_\mu d|D\rangle$ and $\langle U|\bar{u}\gamma_\mu\gamma_5 d|D\rangle$.

The isospin doublet U–D would exhibit a strong interaction with pions, e.g. there would be charge exchange reactions like $\pi^- U \to \pi^0 D$. Relations analogous to the Goldberger–Treimann relations or the Adler–Weissberger relations would be valid.

Furthermore the U or D state can be used in a "gedankenexperiment" as target for lepton scattering experiments. This way the distribution functions of the light quarks u, d can be studied. The heavy quark b would constitute essentially a fixed portion of the momentum of the U or D state. The associated distribution function would essentially be a δ function in x-space. Thus the heavy quark contribution to the total momentum can be disregarded. What is left over is the momentum distribution function of the constituent light quark, which we would like to investigate.

The light quark distribution functions are then given in terms of a scaling parameter x, defined to be the momentum of the quarks, divided by the total momentum of the constituent quark. Thus the variable x varies as usual between zero and one.

The states U and D can be polarized. The simple $SU(6)$ wave function is given by

$$\begin{aligned} U \Uparrow = \frac{1}{\sqrt{3}} &[(b \Uparrow b \Uparrow u \Uparrow) + (b \Uparrow u \Uparrow b \Uparrow) \\ &+ (u \Uparrow b \Uparrow b \Uparrow)]\,. \end{aligned} \tag{2}$$

In QCD the light quark distribution functions are given by the matrix elements of the bilocal densities $\bar{q}(x)\gamma_\mu q(y)$ or $\bar{q}(x)\gamma_\mu\gamma_5 q(y)$ at lightlike distances. Taking these matrix elements, one arrives at the distribution functions $u_+(x), u_-(x), d_+(x)$ and $d_-(x)$ of the U state. The indices + or − denote the helicity + or − of the corresponding quark in a polarized U state with positive helicity.

Let us first denote the sum rules following from the exact flavor conservation laws. The matrix element $\langle U|u^+u|U\rangle$ is, of course, given by one, the matrix element

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$\langle U|d^+ d|U\rangle$ vanishes. Thus we have

$$\int_0^1 (u_+ + u_- - \bar{u}_+ - \bar{u}_-)\,\mathrm{d}x = 1, \tag{3}$$

$$\int_0^1 \left(d_+ + d_- - \bar{d}_+ - \bar{d}_-\right)\mathrm{d}x = 0.$$

The rules above are the analog of the Adler sum rule in the case of nucleons. We proceed to discuss the analog of the Bjorken sum rule denoting the axial coupling constant of the U–D system by g_a:

$$g_a = \int_0^1 \{[(u_+ + \bar{u}_+) - (u_- + \bar{u}_-)] \tag{4}$$
$$- \left[\left(d_+ + \bar{d}_+\right) - \left(d_- + \bar{d}_-\right)\right]\}\,\mathrm{d}x.$$

This sum rule concerns the isotriplet, i.e. the matrix element $\langle U|\bar{u}\gamma_\mu\gamma_5 u_- - \bar{d}\gamma_\mu\gamma_5 d|U\rangle$. We can also consider the matrix element of the isosinglet current $\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d$. The associated sum rule

$$\Sigma = \int_0^1 [(u_+ + \bar{u}_+ - u_- - \bar{u}_-) \tag{5}$$
$$+ \left(d_+ + \bar{d}_+ - d_- - \bar{d}_-\right)]\,\mathrm{d}x$$

gives a number Σ which can be viewed as the contribution of the u- and d-quarks to the U spin.

In a naive model the constituent u-quark inside the U particle would be composed solely of a u-quark with positive helicity, i.e. all density functions vanish except for u_+:

$$\int_0^1 u_+ \mathrm{d}x = 1, \quad g_a = \Sigma = 1,$$

$$d_+ = d_- = \bar{d}_+ = \bar{d}_- = u_- = \bar{u}_+ = 0. \tag{6}$$

This relation would correspond to the $SU(6)$ result in the nucleon: $|G_A/G_V| = 5/3$. In reality we have $|G_A/G_V| = 1.26$, i.e. a reduction from 5/3 by nearly 25%. Taking the same reduction for U, D, as an example, we expect for the U–D system $g_a \cong 0.75$, instead of $g_a = 1$.

The axial-vector coupling constant g_a would fulfill a Goldberger–Treimann relation and would be related to the π–U–D coupling constant. The associated axial-vector current will be conserved in the limit $m_u = m_d = 0$.

However the singlet current $\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d$ is not conserved in this limit due to the gluon anomaly:

$$\partial^\mu \left(\bar{u}\gamma_\mu\gamma_5 U + \bar{d}\gamma_\mu\gamma_5 d\right) = \frac{g^2}{4\pi^2}\cdot\frac{1}{8}\varepsilon_{\alpha\beta\gamma\delta}G_a^{\alpha\beta}G_a^{\gamma\delta} \tag{7}$$

($G_a^{\mu\nu}$ is the gluon field strength).

It is well known that the gluonic anomaly leads to an abnormal mixing pattern for the 0^{-+} mesons, implying a strong violation of the Zweig rule in the 0^{-+} channel. We consider the anomaly as the reason why in the case of the nucleon the axial singlet charge deviates strongly from the naive quark model value [4].

As can be seen directly from the sum rules given above, we obtain immediately the naive result $g_a = \Sigma$, if all d densities vanish. If we take as an example $g_a = 0.75$ and $\bar{u} = d = \bar{d} = 0$, we obtain

$$\int_0^1 (u_+ - u_-)\,\mathrm{d}x = 0.75, \quad \int_0^1 (u_+ + u_-)\,\mathrm{d}x = 1, \tag{8}$$
$$\int_0^1 u_+\mathrm{d}x = 0.875, \quad \int_0^1 u_-\mathrm{d}x = 0.125.$$

In this case 75% of the U spin would be given by the spin of the u-quark; the remaining 25% are due to other effects like orbital effects and gluons.

However in the presence of the QCD anomaly the picture changes since $\Sigma \neq g_a$. We isolate the d-integral and obtain

$$2\int_0^1 \left(d_+ + \bar{d}_+ - d_- - \bar{d}_-\right)\mathrm{d}x = \Sigma - g_a. \tag{9}$$

The difference $\Sigma - g_a$ is given by the matrix element $\langle U|\bar{d}\gamma_\mu\gamma_5 d|U\rangle$ which in a naive picture vanishes. We decompose this into an isosinglet and isotriplet term:

$$\left\langle U|\bar{d}\gamma_\mu\gamma_5 d|U\right\rangle = \frac{1}{2}\left\langle U|\bar{d}\gamma_\mu\gamma_5 d - \bar{u}\gamma_\mu\gamma_5 d|U\right\rangle \tag{10}$$
$$+\frac{1}{2}\left\langle U|\bar{d}\gamma_\mu\gamma_5 d + \bar{u}\gamma_\mu\gamma_5 u|U\right\rangle.$$

The isospin triplet term is determined via a Goldberger–Treimann relation and related to PCAC and the associated pion pole.

Suppose that PCAC would also be valid for the singlet current. In this case there would be a Goldstone particle (the η meson with quark composition $\frac{1}{\sqrt{2}}\left(\bar{u}u + \bar{d}d\right)$, and the π^0 and η contribution would cancel. The matrix element vanishes, and we have $\Sigma = g_a$.

In reality, this is not true. One finds, for example, for the nucleon $\Sigma \approx 0.30$. We set as an illustration $\Sigma = 0.30$ and obtain

$$\int_0^1 \mathrm{d}x \left(d_+ + \bar{d}_+ - d_- - \bar{d}_-\right) = \frac{1}{2}(\Sigma - g_a)$$
$$\cong -0.22, \tag{11}$$
$$\int_0^1 \mathrm{d}x\,(u_+ + \bar{u}_+ - u_- - \bar{u}_-) = \frac{1}{2}(\Sigma + g_a) \cong 0.53.$$

We note that due to the QCD anomaly $\bar{q}q$ pairs are generated inside the U particle. This is a non-perturbative effect, like the QCD anomaly itself. Furthermore the $\bar{q}q$ pairs are polarized, cancelling partially the spin of the

u-quark. Note that the sign of $(\Sigma - g_a)$ is negative. We cannot predict the sign, but we take it from experiment. The sum rule for the d densities above implies that the sum $\left(d_- + \bar{d}_-\right)$ is non-zero, but it does not imply that the sum $\left(d_+ + \bar{d}_+\right)$ is non-zero. Thus $\left(d_+ + \bar{d}_+\right)$ can be zero or very small.

The $(\bar{q}q)$ pairs, generated by the QCD anomaly, e.g. by the gluonic dynamics, are not related to the u-quark directly, and one therefore expects in particular $d_- = \bar{d}_-$. The simplest way to obey the sum rules is $d_+ = \bar{d}_+ = 0$, $d_- = \bar{d}_- = \bar{u}_-$.

In this case a polarized constituent u-quark is dominated by the u_+ function, accompanied by $\left(\bar{d}d\right)$– and $(\bar{u}u)$ pairs, which partially cancel the spin of the u-quark.

An interesting case, probably close to reality, is

$$\bar{u}_+ = 0, \quad d_+ = \bar{d}_+ = 0, \tag{12}$$

$$\int_0^1 \left(d_- + \bar{d}_-\right) \mathrm{d}x = -\frac{1}{2} (\Sigma - g_a) \cong 0.22,$$

$$\int_0^1 \bar{u}_- \mathrm{d}x = - (\Sigma - g_a) \cong 0.11,$$

$$\frac{1}{2} (\Sigma + g_a) = \int_0^1 \mathrm{d}x \, (u_+ - u_- - \bar{u}_-) \cong 0.53. \tag{13}$$

Now we include the s-quark. At first we consider the case of $SU(3)$-symmetry: $m_u = m_d = m_s$. The total spin sum is given by

$$\Sigma = \int_0^1 [(u_+ + \bar{u}_+ - u_- - \bar{u}_-) + (d_+ + \bar{d}_+ - d_- - \bar{d}_-) + (s_+ + \bar{s}_+ - s_- - \bar{s}_-)] \, \mathrm{d}x. \tag{14}$$

We obtain

$$\Sigma - g_a = \int_0^1 \left[\left(d_+ + \bar{d}_+ - d_- - \bar{d}_-\right) 2 + (s_+ + \bar{s}_+ - s_- - \bar{s}_-)\right] \mathrm{d}x. \tag{15}$$

Again we set $s_+ = \bar{s}_+ = d_+ = \bar{d}_+ = 0$. Furthermore we have $d_- = \bar{d}_- = s_- = \bar{s}_-$ and obtain

$$\int_0^1 \left(d_- + \bar{d}_-\right) \mathrm{d}x = \int_0^1 [(s_- + \bar{s}_-)] \, \mathrm{d}x \cong 0.15,$$

$$\int_0^1 \bar{u}_- \mathrm{d}x \cong 0.075. \tag{16}$$

Again the spin is partially cancelled by the $\bar{q}q$ pairs, this time including $\bar{s}s$ pairs.

Now we consider the realistic case with $SU(3)$-breaking. It is well known that the physical wave function of the η meson and the η' meson are approximately given by

$$\eta = \frac{1}{2}\left(\bar{u}u + \bar{d}d - \sqrt{2}\bar{s}s\right), \quad \eta' = \frac{1}{2}\left(\bar{u}u + \bar{d}d + \sqrt{2}\bar{s}s\right). \tag{17}$$

Thus in reality we are between the two cases discussed above. As an example, which is probably close to reality, we take

$$\int_0^1 \left(d_- + \bar{d}_-\right) \mathrm{d}x = 0.18,$$

$$\int_0^1 \bar{u}_- \mathrm{d}x \cong 0.09,$$

$$\int_0^1 (s_- + \bar{s}_-) \, \mathrm{d}x = 0.11. \tag{18}$$

Again we see that the QCD anomaly is the reason why the spin is partially cancelled by the $\bar{q}q$ pairs, although the $\bar{s}s$ pairs are less relevant than the $\bar{d}d$ and $\bar{u}u$ pairs.

We think that we have found in the QCD anomaly the reason why the spin of the nucleon is a rather complicated object. The spin is reduced by $\bar{q}q$ pairs, which partially cancel the spin of the constituent quarks. The $\bar{q}q$ pairs are polarized. This polarization can be observed in lepton–nucleon scattering.

The question arises who carries the remaining part of the spin. The departure of $|G_A/G_V|$ from 5/3 indicates that orbital effects are there. They make up about 25% of the spin of the nucleon. The remaining part of about 45% is related to the QCD anomaly. Since the latter is a $\bar{q}q$ effect, we conclude that about 45% of the nucleon spin is carried by gluons. We summarize: 30% of the spin is carried by the valence quarks and the $\bar{q}q$ pairs, 25% by orbital effects, 45% by gluons.

The polarized $\bar{q}q$ pairs should be searched for in the experiments. Also the gluonic contribution can be observed in the experiments, especially by studying the $\bar{c}c$ production in lepton–nucleon scattering, as done in the Compass experiment [4].

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